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CLAN STRUCTURE ANALYSIS AND RAPIDITY GAP PROBABILITY

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Clan structure analysis in rapidity intervals is generalized from negative binomial multiplicity distribution to the wide class of compound Poisson distributions. The link of generalized clan structure analysis with correlation functions is also established. These theoretical results are then applied to minimum bias events and evidentiate new interesting features, which can be inspiring and useful in order to discuss data on rapidity gap probability at TEVATRON and HERA.

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Introduction

Clan structure analysis puzzled experts since its introduction in multiparticle dynamics. The real question is: is clan structure analysis simply a new parametrization or, in view of the regularities which it reveals in different classes of reactions, it has a deeper physical insight? In [1] it has been shown that the inverse of the average number of particles per clan is exactly the *void scaling function*, which was introduced in order to test hierarchical structure of correlations. The behavior of the average number of particles per clan provides therefore information on the structure of correlations functions in multiparticle production. In this paper, starting from some ideas developed in [1], we show that also the average number of clans in a region of phase space has an suggestive physical meaning: it is simply linked to the probability to detect no particles in that region, as well as to the normalized factorial cumulants generating function in the same region. The average number of clans provides therefore information both on rapidity gap probabilities and on the general features of correlation functions. It is interesting to remark that these properties are not linked to the distribution which motivated clan structure analysis, *i.e.*, Negative Binomial (NB) Multiplicity Distribution (MD), but are common to the whole class of Compound Poisson Distributions (CPD's) (or discrete infinitely divisible distributions) to which NB MD belongs.

In Section 1 we generalize clan structure analysis to CPD's and discuss the theorems which establish the above mentioned connections. In Section 2 we apply these theorems to the domain of validity of NB regularity. Interesting new features are revealed and in particular the energy independence of the rapidity gap probability in different classes of reactions as well as its leveling in large rapidity intervals.

I. Generalized clan structure analysis, correlations and rapidity gap probability

A CPD is fully determined by its generating function, $f_{CPD}(z)$, and is described in general by the following equation

$$f_{CPD}(z) = e^{\bar{N}_{\text{g-clan}}[g(z)-1]} \quad (1)$$

where $\bar{N}_{\text{g-clan}}$ is the average number of independent intermediate objects generated according to a Poisson distribution; they have been called generalized clans (g-clans) in [1] whereas $g(z)$ is the particle generating function for an average g-clan. Notice that a physical process described by a CPD is a typical two steps

process: intermediate independent g-clans produced in the first step decay into final charged particles in the second step following the MD corresponding to the generating function $g(z)$; g-clans are indeed groups of particles of common origin and each g-clan contains at least one particle, *i.e.*, according to this definition, particles MD of an average g-clan has to be truncated:

$$g(z)|_{z=0} = 0 \quad (2)$$

This description fits the full production process, and therefore applies directly to full phase space analysis. When examining a rapidity interval Δy one should understand that we are not defining a new process but we are again dealing with the g-clans and particles described above. Thus one is lead again to eq. (1) but now $g(z)$ becomes $g(z; \Delta y)$, *i.e.*, it is the generating function of the MD of one g-clan with respect to the interval Δy , which is in general different for different intervals:

$$f(z; \Delta y) = e^{\bar{N}_{\text{g-clan}} [g(z; \Delta y) - 1]} \quad , \quad g(z; \Delta y)|_{z=0} \equiv q_0(\Delta y) \neq 0 \quad (3)$$

In fact $q_0(\Delta y)$ is the probability that a g-clan does not produce any particle within the interval Δy , which is indeed not zero. It is easily seen that by defining a new generating function $\tilde{g}(z; \Delta y)$:

$$\tilde{g}(z; \Delta y) = \frac{g(z; \Delta y) - q_0(\Delta y)}{1 - q_0(\Delta y)} \quad (4)$$

one can write eq. (3) as

$$f(z; \Delta y) = e^{\bar{N}_{\text{g-clan}}(\Delta y) [\tilde{g}(z; \Delta y) - 1]} \quad , \quad \tilde{g}(z; \Delta y)|_{z=0} = 0 \quad (5)$$

Since now $\tilde{g}(z; \Delta y)|_{z=0} = 0$, eq. (5) involves only those g-clans which produce at least one particle in the interval Δy . (It should be pointed out that the form of the distribution used in the standard clan analysis of data is indeed eq. (5).)

Finally note that eq.s (3) and (5) are linked by binomial convolution at *g-clan level*, since one finds

$$\bar{N}_{\text{g-clan}}(\Delta y) = \bar{N}_{\text{g-clan}} [1 - q_0(\Delta y)] \quad (6)$$

which, in terms of probabilities for the observation of N' g-clans in the interval Δy , $p_{N'}(\Delta y)$, and of N g-clans in full phase space, p_N , corresponds to

$$p_{N'}(\Delta y) = \sum_{N=N'}^{\infty} \binom{N}{N'} q_0(\Delta y)^{N-N'} [1 - q_0(\Delta y)]^{N'} p_N \quad (7)$$

Eq. (6) should be contrasted with the application of binomial convolution at particle level, which leads[2], in the case of NBD, to

$$\bar{n}(\Delta y) = \bar{n} [1 - P_0(\Delta y)] \quad k(\Delta y) = k$$

in sharp contrast to experimental data; on the contrary, eq.s (6) and (7) don't give any relationship between parameters at particle level, since they don't imply any relationship between $g(z)$ and $g(z; \Delta y)$.

It is well known that a NB MD can be obtained by requiring a logarithmic distribution of particles inside an average g-clan. NB MD is an appreciated two-parameter MD in multiparticle dynamics; its standard parameters are the average number of charged particle, \bar{n} , and the parameter k^{-1} , which is linked to the dispersion $D^2 = \bar{n}^2 - \bar{n}^2$ by the relation $k^{-1} = D^2/\bar{n}^2 - 1/\bar{n} = \kappa_2$ (κ_2 is the second-order normalized factorial cumulant). NB distribution has been proposed since 1972[3] with success for describing experimental data on final charged particle MD's in full phase space and later on[4] in symmetric rapidity intervals in hadron-hadron collisions, and then extended to all classes of high energy reactions (deep inelastic scattering, e^+e^- annihilation and AA collisions) [5]. The interest on the class of CPD's, to which NB belongs, seems therefore fully justified. It should be added that the interpretation of NB regularity[6] led to analyze experimental data in terms of the average number of clans, \bar{N} (the name clan was introduced in this framework) and of the average number of particles per clan \bar{n}_c (clan structure analysis). The new parametrization is linked to the old one in terms of \bar{n} and k by the following equations:

$$\bar{N} = k \log \left(1 + \frac{\bar{n}}{k} \right) \quad (8)$$

$$\bar{n}_c = \frac{\bar{n}}{\bar{N}} \quad (9)$$

The interest here is on the connection of the generalized clan properties with the probability to detect no particles in a rapidity interval Δy for a generic CPD; the general theory will be applied to NB MD in the next section. Being in eq.s (1) and (5) the generating function $g(z)$ not a priori specified, clan concept results to be much more general than the standard one defined by eq.s (8) and (9). Depending on the choice of $g(z)$ one has in fact different CPD generating functions $f_{CPD}(z)$. Table 1 shows, in addition to NB MD, two other CPD's frequently discussed in the literature[7], *i.e.*, the composition of a Poisson with a truncated Poisson distribution (Thomas distribution[8]) and the composition of a Poisson with a truncated

geometric distribution (Pólya-Aeppli[9]). All these are two-parameter distributions. Notice that they can be obtained as limiting forms of the composition of a Poisson with a NB distribution, *i.e.*, a three parameters distribution. A fourth distribution is shown in Table 1 as a typical example of a three parameter distribution, the Partially Coherent Laser Distribution (PCLD)[10]: its generating function is the product of the generating functions of NB and Pólya-Aeppli distributions; accordingly, the PCLD belongs to the class of CPD's, a fact which has been overlooked in the literature.

It should be noticed that the belonging to the class of CPD's for a MD can be tested either by the sign of the corresponding combinants*[11] or, for two-parameter distributions only, by the validity of Linked-Pair Ansatz (LPA) for the corresponding n -particle correlations functions[12] (its violation for two-parameters MD's implies that the MD is not a CPD).

The study of $\bar{N}_{\text{g-clan}}(\Delta y)$ and $P_0(\Delta y)$ for the class of CPD's is based on the following simple theorems.

Theorem 1. Being by definition

$$P_0(\Delta y) \equiv f(z; \Delta y)|_{z=0} \quad (10)$$

from eq. (5) it follows that

$$P_0(\Delta y) = e^{-\bar{N}_{\text{g-clan}}(\Delta y)} \quad (11)$$

Eq. (11) says that for CPD's the average number of g-clans in the rapidity interval Δy determines the probability of detecting no particles in the same interval.

Theorem 2. From the expression of the generating function in terms of normalized factorial cumulants[7]

$$f(z; \Delta y) = \exp \left\{ \sum_{n=1}^{\infty} \frac{\kappa_n(\Delta y)}{n!} [\bar{n}(\Delta y)(z-1)]^n \right\} \quad (12)$$

a second powerful theorem on $P_0(\Delta y)$ can be proved via eq. (10). It establishes the link of $P_0(\Delta y)$ with the corresponding normalized factorial cumulants in the interval Δy , $\kappa_n(\Delta y)$. One has in fact

$$P_0(\Delta y) = \exp \left[\sum_{n=1}^{\infty} \frac{[-\bar{n}(\Delta y)]^n}{n!} \kappa_n(\Delta y) \right] \quad (13)$$

* Combinants C_n are defined by $f(z) = \exp[\sum_{n=0}^{\infty} C_n(z^n - 1)]$ such that for CPD's one has $\bar{N} = -\log P_0 = \sum_{n=0}^{\infty} C_n$ and C_n/\bar{N} is the probability to have n particles inside an average g-clan; therefore one has a CPD *iff* $P_0 > 0$ and $C_n \geq 0, \forall n$.

where normalized factorial cumulants $\kappa_n(\Delta y)$ are n -fold integrals of the normalized correlation functions $c_n(y_1, \dots, y_n)$:

$$\kappa_n(\Delta y) = \int_{\Delta y} dy_1 \dots \int_{\Delta y} dy_n c_n(y_1, \dots, y_n) \quad (14)$$

$P_0(\Delta y)$ turns out to be determined according to eq.s (13) and (14) by the sum of all n -order normalized factorial cumulants or, equivalently, by the integrals over the interval Δy of corresponding n -particle correlation functions. Notice that the exponent in eq. (13) is the normalized factorial cumulant generating function.

Theorem 3. Finally it can be proved just by inspection of eq.s (11) and (13) that

$$\bar{N}_{\text{g-clan}}(\Delta y) = - \sum_{n=1}^{\infty} \frac{[-\bar{n}(\Delta y)]^n}{n!} \kappa_n(\Delta y) \quad (15)$$

i.e., the average number of g-clans for a CPD in a given rapidity interval Δy can be obtained by calculating the normalized factorial cumulants generating function in the same interval and vice versa the normalized factorial cumulant generating function is fully determined by the average number of g-clans.

The generality of the above mentioned theorems leads to striking results when applied to the class of hierarchical models[13], *i.e.*, to the class of models in which $c_n(y_1, \dots, y_n)$ can be expressed as the product of $(n - 1)$ two-particle correlation functions or, in terms of normalized factorial cumulants:

$$\kappa_n(\Delta y) = A_n [\kappa_2(\Delta y)]^{n-1} \quad (16)$$

A_n in eq. (16) is independent of the energy and rapidity interval considered, but it depends on the different choices of the generating function $g(z; \Delta y)$ in eq. (5). Distributions NB, Thomas and Pólya-Aeppli, being two-parameter CPD's, differently from PCLD, satisfy all eq. (16), with the corresponding A_n coefficients shown in Table 1. Accordingly, for hierarchical models one has

$$P_0(\Delta y) = \exp \left[\sum_{n=1}^{\infty} A_n \frac{[-\bar{n}(\Delta y)]^n}{n!} [\kappa_2(\Delta y)]^{n-1} \right] \quad (17)$$

with

$$\kappa_2(\Delta y) = \int_{\Delta y} dy_1 \int_{\Delta y} dy_2 c_2(y_1, y_2) \quad (18)$$

As already pointed out in the Introduction, these results represent the counterpart in terms of the average number of g-clans of the properties of the void scaling function $\mathcal{V}(\Delta y)$ discussed in [1,14,15] in order to test normalized factorial cumulants hierarchical structure. Apparently the probability to detect no particles for

a CPD in a given rapidity interval is controlled by and controls both the average number of g-clans of the full MD and the normalized factorial cumulant generating function in the same interval.

These results have a possible explanation in the existing connection between the n -particle and zero-particle probabilities as given by the following equation

$$P_n(\Delta y) = \frac{[-\bar{n}(\Delta y)]^n}{n!} \frac{\partial^n P_0(\Delta y)}{\partial \bar{n}(\Delta y)^n} \quad (19)$$

which can be obtained by allowing only $\bar{n}(\Delta y)$ to vary in $P_0(\Delta y)$ (all the other parameters of $P_0(\Delta y)$ are taken fixed with respect to the $\bar{n}(\Delta y)$ variation). It is to be noticed that from eq. (19) one can deduce the following differential equation

$$(n+1)P_{n+1}(\Delta y) - nP_n(\Delta y) = -\bar{n}(\Delta y) \frac{\partial P_n(\Delta y)}{\partial \bar{n}(\Delta y)} \quad (20)$$

or, in terms of the corresponding generating function, $f(z; \Delta y)$,

$$\bar{n}(\Delta y) \frac{\partial f(z; \Delta y)}{\partial \bar{n}(\Delta y)} = (z-1) \frac{\partial f(z; \Delta y)}{\partial z} \quad (21)$$

i.e., $f(z; \Delta y)$ depends on z and $\bar{n}(\Delta y)$ through the product $\bar{n}(\Delta y)(z-1)$ only.

From eq. (12) all distributions whose normalized factorial cumulants do not depend on the average multiplicity, like for instance NB MD, satisfy the above property (19). It is interesting to remark that eq. (19) is fulfilled by many distributions used in literature, like Poisson, NB, Pólya-Aeppli, Thomas, and all distributions which can be written as a positive weight superposition of Poisson distributions[16] (Poisson transforms of a continuous distribution).

The fact that eq (11) holds for any MD belonging to the class of CPD's, including NB MD, is of particular relevance. The importance of the result is enhanced by remembering the definition of void scaling function $\mathcal{V}(\Delta y)$ (see [1]), which is just the inverse of the average number of particles per g-clan, *i.e.*,

$$\mathcal{V}(\Delta y) = \frac{\bar{N}_{\text{g-clan}}(\Delta y)}{\bar{n}(\Delta y)} = \frac{1}{\bar{n}_{\text{c,g-clan}}(\Delta y)} \quad (22)$$

For the NB MD notice that $\bar{N}_{\text{g-clan}}(\Delta y)$ and $\bar{n}_{\text{c,g-clan}}(\Delta y)$ of eq.s (11) and (22) coincide with eq. (8) and (9).

Altogether above mentioned formulae show how deep and intriguing is the meaning of what was believed for long time just a new parametrization of MD's for interpreting NB regularity; generalized clan structure analysis turns out to be in general the analysis of voids or gaps properties in phase space and of the n -particle correlation function structure of the corresponding MD's.

II. Rapidity gap probability from experimental multiplicity distributions

It has been shown by Cugnon and Harouna[17] that goodness of fits to experimental data on final charged particle MD's in terms of CPD's does not change much for different choices of the generating function $g(z)$. More precisely, all these fits are comparable with a NB fit. This fact can be interpreted as an indication that what matters more in the above mentioned context is the CPD nature of the process (a two steps process) than the detailed structure of the MD obtained by fixing the generating function $g(z)$. In view of this remark, limiting the discussion just to one MD does not restrict the domain of validity of our general conclusions: they refer in an approximate sense – with the appropriate warnings – to the whole class of CPD's. The most natural choice is to discuss the NB MD. It is in fact quite clear that when one wants to apply results on CPD to the real world one meets necessarily just this distribution, whose approximate validity for describing final particles MD's is well established. The interest of the application of results of Section I to the NB MD lies in the fact that our results should be thought common, in view of previous remark, to the whole class of CPD's.

Deviations from NB behavior in symmetric rapidity intervals were indeed observed in $\bar{p}p$ reactions at c.m. energy $\sqrt{s} = 900$ GeV and at c.m. energy $\sqrt{s} = 1800$ GeV as in e^+e^- annihilation at LEP (c.m. energy $\sqrt{s} = 91$ GeV). A typical shoulder structure is seen in all these experiments. Shoulder effect is understood[18] in e^+e^- annihilation as the superposition of MD's of events of different topologies and each topology satisfies well NB behavior (see [19] for a critical discussion of this point). In $\bar{p}p$ it has been proposed[20] to describe the effect by the superposition of two NB MD's, which is justified by the onset of a semi-hard component. The success of the fit is of course weakened in this case by the large number of parameters introduced. Notice that the analysis of the topology of the events is here not possible since UA5 Collaboration cannot measure particles' momenta. From our point of view, it is to be remarked that a linear superposition of two or more CPD's allow to study the probability of detecting no particles in a given rapidity interval in terms of generalized clan structure analysis in the same interval. In fact eq. (5) can be generalized as follows

$$f(z; \Delta y) = \alpha f^{(1)}(z; \Delta y) + (1 - \alpha) f^{(2)}(z; \Delta y) \quad (23)$$

with α a parameter controlling the relative weight of one distribution to the other. Accordingly, from eq. (10), one obtains

$$P_0(\Delta y) = \alpha e^{-\bar{N}_{\text{g-clan}}^{(1)}(\Delta y)} + (1 - \alpha) e^{-\bar{N}_{\text{g-clan}}^{(2)}(\Delta y)} \quad (24)$$

Consequently, observed deviations from NB behavior can be interpreted again in the same framework of CPD's. Shoulder effect in rapidity intervals could be analyzed in such terms also at TEVATRON.

Let us now examine $P_0(\Delta y)$ properties in the domain of validity of NB regularity. This analysis can be considered an example of the result discussed previously for CPD's as well as an alternative procedure for studying rapidity gap probabilities as they appear in the real world.

In Figure 1 the rapidity gap probability in symmetric rapidity interval Δy , $P_0(\Delta y)$, obtained by performing a NB fit on the MD, is shown as a function of the width of the rapidity interval Δy for hh collisions at different c.m. energies ranging from $\sqrt{s} = 22$ GeV[21] to $\sqrt{s} = 546$ GeV[4]. The rapidity gap probability decreases almost linearly for small rapidity intervals and then levels for larger rapidity intervals; this last behavior corresponds to the well-known bending of the average number of clans observed experimentally at the border of phase space. This result has been interpreted in [6] as the effect of conservation laws, which become important at the boundary of phase space; this picture is supported by the experimental behavior of the average number of particles per clan, which after a quick increase reaches a maximum and then decreases in large rapidity intervals. It is to be pointed out that the rapidity gap probability in hh collisions is approximately energy independent from $\sqrt{s} = 22$ GeV up to $\sqrt{s} = 546$ GeV, as of course it is to be expected from the corresponding behavior in clan structure analysis.

A preliminary analysis of minimum bias events at TEVATRON (at c.m. energy $\sqrt{s} = 1800$ GeV) has been actually performed by CDF Collaboration[22]: it has been found that at this c.m. energy the shoulder structure becomes important not only in large rapidity intervals as it was found at c.m. energy $\sqrt{s} = 900$ GeV[23], but also in smaller rapidity intervals. It could be interesting to investigate if this structure could be still explained in terms of the superposition of two different MD's, each of them of CPD type (for instance two NB MD's). In this case, as previously discussed, one would still be able to exploit CPD structure to determine $P_0(\Delta y)$ properties from the full MD and to compare this behavior with that shown in Figure 1.

A similar trend for the rapidity gap probability has been observed by DØ Collaboration[24] for a different sample of events, *i.e.*, for dijet events with transverse energy of each jet greater than 30 GeV. This qualitative common structure is very remarkable since in this second case the selected process is a hard one.

Notice that the use of CPD properties for studying the rapidity gap probability

in minimum bias events at TEVATRON would be really helpful for a deeper understanding of $D\emptyset$ results; in fact, this analysis could provide the estimate of the rapidity gap probability expected from the ordinary soft gluon radiation, *i.e.*, the estimate of the soft background contribution which is not under control so far in $D\emptyset$ data and can mask the detection of a hard production mechanisms, like, for instance, hard pomeron exchange[25].

In view of the qualitative agreement between minimum bias results obtained in the framework of CPD properties (and in particular NB MD) and $D\emptyset$ results for dijet events, it could be natural to assume that the resulting MD is of CPD type also for dijet events. Accordingly, one can determine $P_0(\Delta y)$ in dijet events not only by looking directly at regions of phase space without any particle, but also, with a completely independent method, by applying clan structure analysis to the full MD. Notice that this second method should decrease the statistical error on the rapidity gap probability, since it is based on the analysis of the full sample of dijet events.

The structure of rapidity gap probability similar to that observed in hh collisions is seen also in deep inelastic scattering[26] (see Figure 2). It is interesting to remind[6] that in this case the average number of clans has the same behavior as in hh collisions, but the average number of particles per clan has a behavior similar to e^+e^- annihilation, *i.e.*, clans in deep inelastic scattering are much smaller than in hh collisions. This property could be relevant for the interpretation of the recent experimental result found at HERA[27], where an excess of events with a large rapidity gap and small multiplicity has been observed.

Particular attention should be paid to e^+e^- annihilation where, as well known, the average number of clans has a different slope with respect to the behavior shown in Figure 1 and 2. Therefore, one should expect a steeper slope for $P_0(\Delta y)$. This is shown in Figure 3. This analysis is here limited to two-jet events in order to make possible the comparison between HRS[28] and DELPHI[18] data. The study of rapidity gap probability is here simplified with respect to reactions with hadrons in the initial state, because in e^+e^- annihilation a gap cannot be filled by particles produced by the fragmentation of the initial state remnants; this reaction constitutes therefore a “clean” environment to study the physics of rapidity gaps. In fact the information contained in Figure 3 can be used to estimate the two-jets events contribution to the background in the framework of the search proposed in [29].

Above results can be interpreted according to Theorems 2 and 3 discussed in Section 1 also in terms of correlations properties. The effect of correlations can

be tested indeed by looking at the difference between the observed rapidity gap probability $P_0(\Delta y)$, *i.e.*, eq. (11) with $\bar{N}(\Delta y)$ given by eq. (8) as requested by NB behavior, and the rapidity gap probability corresponding to independent particle production, $e^{-\bar{n}(\Delta y)}$, *i.e.*, to a Poissonian distribution with the same average number of charged particles $\bar{n}(\Delta y)$, to which $\bar{N}(\Delta y)$ reduces in this case.

In Table 2 experimental results for $e^{-\bar{N}(\Delta y)}$ and $e^{-\bar{n}(\Delta y)}$ for different reactions in two fixed rapidity intervals are shown: the effect of correlations is remarkable in hh collisions where the rapidity gap probability is larger by many order of magnitude with respect to the hypothetical Poissonian behavior; the difference is narrower in e^+e^- annihilation two-jets events, confirming the quasi-Poissonian behavior of MD's. Notice that the direct comparison between hh collisions and e^+e^- annihilation two-jets events shows that the rapidity gap probability is larger in hh collisions than in e^+e^- annihilation. This result agrees with previous interpretation of the parameter $1/k(\Delta y)$ of NB MD as an aggregation parameter[6]:

$$\frac{1}{k} = \frac{\mathcal{P}(n=2, N=1)}{\mathcal{P}(n=2, N=2)} \quad (25)$$

($\mathcal{P}(n, N)$ is the probability to produce n particles distributed in N clans), as well as with the relation of $1/k(\Delta y)$ with two-particle correlation function (see eq. (18)). In fact, suppose we compare two different reactions having the same average number of particles in a given rapidity interval Δy ; it is clear that to larger rapidity gap probability corresponds at fixed number of particles more aggregation among final particles, *i.e.*, larger values of $1/k(\Delta y)$ as can be seen just by inspection of eq.s (11) and (8). At the same time larger aggregation corresponds to larger two-particle correlations as can be noticed again just by inspection of eq. (18).

Conclusions

The parameters introduced some time ago by Léon Van Hove and one of the present authors in order to interpret the wide occurrence in all classes of reactions of NB regularity have been found to possess a deep and intriguing physical meaning; they are related to n -particle correlation functions and can be used to test their eventual hierarchical structure, as it has been already anticipated in part in our previous work on void scaling function. This finding is shown to be common to all classes of CPD's, where the concept of generalized average number of clans, $\bar{N}_{\text{g-clan}}$, and of average number of particles per g-clan, $\bar{n}_{\text{c,g-clan}}$, can be defined. This fact is of particular relevance: it points out the two steps nature of the physical process under investigation, which seems to be not much influenced

by the detailed structure of the MD of particles inside an average g-clan, *i.e.*, by the structure of the second step in the process.

The paper should be considered a contribution to the integrated description of n -particles correlation function and MD's in rapidity intervals. The link is represented by the probability to detect no particles in different rapidity regions of phase space. Accordingly an alternative approach is proposed in order to determine rapidity gap probability in terms of the average number of clans in the rapidity interval considered.

The detailed study of rapidity gap probability in the domain of validity of NB regularity reveals interesting features. In particular, one should mention the energy independence of rapidity gap probability for each class of reactions and its leveling for large rapidity intervals in hh and lh collisions. These remarks can be useful and inspiring in order to discuss data on rapidity gap probability at TEVATRON and HERA, where usually a sample of events different from minimum bias is selected.

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Figure Captions

Fig. 1: Rapidity gap probability $P_0(\Delta y)$ as a function of the rapidity width Δy obtained from NB fits in hh collisions at different c.m. energies $\sqrt{s} = 22$ GeV, $\sqrt{s} = 200$ GeV and $\sqrt{s} = 546$ GeV.

Fig. 2: Rapidity gap probability $P_0(\Delta y)$ as a function of the rapidity width Δy obtained from NB fits in deep inelastic scattering for various intervals of the total hadronic energy W as indicated in the Figure.

Fig. 3: Rapidity gap probability $P_0(\Delta y)$ as a function of the rapidity width Δy obtained from NB fits to the sample of two-jets events in e^+e^- annihilation at c.m. energies $\sqrt{s} = 29$ GeV and $\sqrt{s} = 91$ GeV.

Table Captions

Tab. 1: generating function $f(z)$ of final particles MD, average number of g-clans, $\bar{N}_{\text{g-clan}}$, and generating function of particles MD inside an average g-clan, $g(z)$, for NB, Thomas, Pólya-Aeppli distributions as a function of the average number of particles \bar{n} and the second-order normalized factorial cumulant κ_2 of each distribution. In the last column A_n coefficients (see eq. (16)) of the above mentioned distributions are indicated. Generating function of the MD, average number of g-clans and generating function of the MD inside a g-clan for the PCLD are also shown in terms of its three parameters A, B, C . It should be added that eq. (16) is not valid in this case.

Tab. 2: Comparison in two different rapidity intervals of rapidity gap probability obtained via the average number of clans from a NB fit, $\exp(-\bar{N})$, and the rapidity gap probability expected for a Poissonian distribution of the same average number of particles, $\exp(-\bar{n})$, for the reactions indicated in the Table. Values of the average number of clans \bar{N} and of the average multiplicity \bar{n} in the same intervals for the same reactions are also shown.

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Table 1

	$f_{CPD}(z)$	$\bar{N}_{\text{g-clan}}$	$g(z)$	A_n
NB	$[1 - \bar{n}\kappa_2(z-1)]^{-1/\kappa_2}$	$\frac{1}{\kappa_2} \log(1 + \bar{n}\kappa_2)$	$\frac{\log(1-bz)}{\log(1-b)}, \quad b = \frac{\bar{n}\kappa_2}{\bar{n}\kappa_2 + 1}$	$(n-1)!$
Thomas	$e^{\frac{1}{\kappa_2}[e^{\bar{n}\kappa_2(z-1)} - 1]}$	$\frac{1}{\kappa_2}(1 - e^{-\bar{n}\kappa_2})$	$\frac{e^{\bar{n}\kappa_2 z} - 1}{e^{\bar{n}\kappa_2} - 1}$	1
Pólya-Aeppli	$e^{\frac{2\bar{n}(z-1)}{2 - \bar{n}\kappa_2(z-1)}}$	$\frac{2\bar{n}}{2 + \bar{n}\kappa_2}$	$\frac{z(1-b')}{1-b'z}, \quad b' = \frac{\bar{n}\kappa_2}{\bar{n}\kappa_2 + 2}$	$\frac{n!}{2^{n-1}}$
PCLD	$e^{\frac{A(z-1)}{1-Cz} \left(\frac{1-C}{1-Cz} \right)^{\frac{B}{\log(1-C)}}}$	$A + B$	$\frac{1}{A+B} \left[A \frac{z(1-C)}{1-Cz} + B \frac{\log(1-Cz)}{\log(1-C)} \right]$	—

Table 2

Rapidity interval: $ y \leq 1.0$				
Reaction	\bar{N}	\bar{n}	$\exp(-\bar{N})$	$\exp(-\bar{n})$
$\bar{p}p$ 900 GeV	2.85 ± 0.05	7.4 ± 0.1	$(5.8 \pm 0.3)10^{-2}$	$(6.0 \pm 0.2)10^{-4}$
e^+e^- 2 jet 29 GeV	3.58 ± 0.12	4.26 ± 0.16	$(2.79 \pm 0.33)10^{-2}$	$(1.4 \pm 0.2)10^{-2}$
e^+e^- 2 jet 91 GeV	3.45 ± 0.13	4.35 ± 0.44	$(3.17 \pm 0.41)10^{-2}$	$(1.3 \pm 0.2)10^{-2}$
Rapidity interval: $ y \leq 1.5$				
Reaction	\bar{N}	\bar{n}	$\exp(-\bar{N})$	$\exp(-\bar{n})$
$\bar{p}p$ 900 GeV	3.57 ± 0.07	11.1 ± 0.1	$(2.8 \pm 0.2)10^{-2}$	$(1.5 \pm 0.2)10^{-5}$
e^+e^- 2 jet 29 GeV	5.56 ± 0.19	6.65 ± 0.25	$(3.8 \pm 0.7)10^{-3}$	$(1.3 \pm 0.3)10^{-3}$
e^+e^- 2 jet 91 GeV	5.06 ± 0.12	7.08 ± 0.47	$(6.3 \pm 0.7)10^{-3}$	$(0.8 \pm 0.4)10^{-3}$

Figure 1

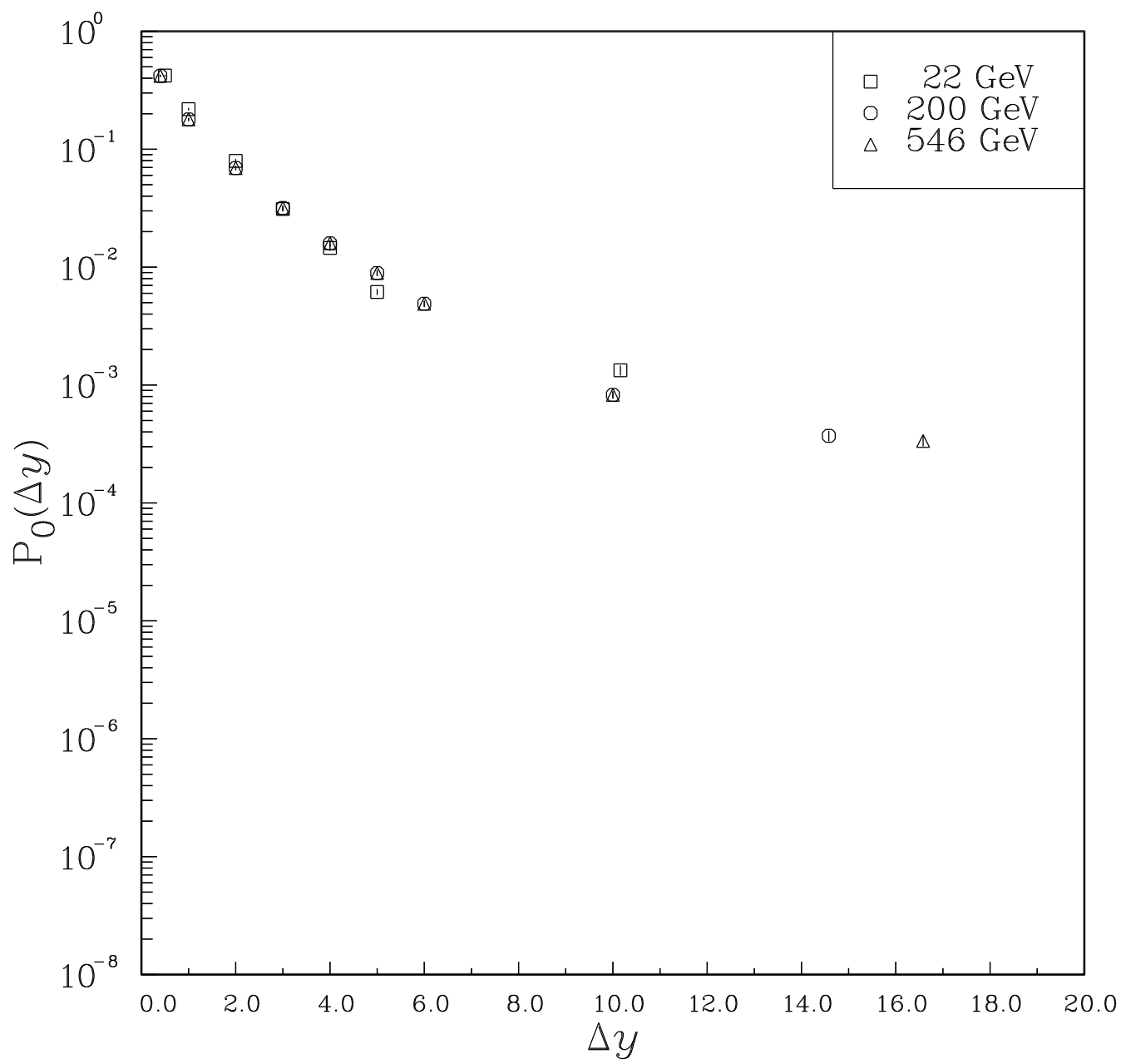


Figure 3

